List MF26 (Partial)   Mathematics 9758

PURE MATHEMATICS

Algebraic series

Binomial expansion :

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + b^n,$$ where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$.

Maclaurin expansion :

$$\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^n}{n!}\mathrm{f}^{(n)}(0) + \ldots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x|<1)$$

$$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \qquad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \qquad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \qquad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1} x^r}{r} + \ldots \qquad (-1 < x \le 1)$$

Partial fractions decomposition

Non-repeated linear factors :

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

Repeated linear factors :

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor :

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2\sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin P + \sin Q \equiv 2\sin \tfrac{1}{2}(P+Q)\cos \tfrac{1}{2}(P-Q)$$

$$\sin P - \sin Q \equiv 2\cos \tfrac{1}{2}(P+Q)\sin \tfrac{1}{2}(P-Q)$$

$$\cos P + \cos Q \equiv 2\cos \tfrac{1}{2}(P+Q)\cos \tfrac{1}{2}(P-Q)$$

$$\cos P - \cos Q \equiv -2\sin \tfrac{1}{2}(P+Q)\sin \tfrac{1}{2}(P-Q)$$

Principal values:

$$-\frac{1}{2}\pi \le \sin^{-1} x \le \frac{1}{2}\pi \qquad (|x| \le 1)$$

$$0 \le \cos^{-1} x \le \pi \qquad (|x| \le 1)$$

$$-\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

Derivatives

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$	
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$	
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$	$\left(\lvert x\rvert < a\right)$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{x-a}{x+a}\right)$	$(x > a)$
$\dfrac{1}{a^2-x^2}$	$\dfrac{1}{2a}\ln\left(\dfrac{a+x}{a-x}\right)$	$\left(\lvert x\rvert < a\right)$
$\tan x$	$\ln(\sec x)$	$\left(\lvert x\rvert < \dfrac{1}{2}\pi\right)$
$\cot x$	$\ln(\sin x)$	$(0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$(0 < x < \pi)$
$\sec x$	$\ln(\sec x + \tan x)$	$\left(\lvert x\rvert < \dfrac{1}{2}\pi\right)$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\dfrac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product:

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

PROBABILITY AND STATISTICS

Standard discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\mathrm{Po}(\lambda)$	$e^{-\lambda}\dfrac{\lambda^x}{x!}$	λ	λ
Geometric $\mathrm{Geo}(p)$	$(1-p)^{x-1} p$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$

Standard continuous distribution

Distribution of X	p.d.f.	Mean	Variance
Exponential	$\lambda e^{-\lambda x}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$

Sampling and testing

Unbiased estimate of population variance :

$$s^2 = \frac{n}{n-1}\left(\frac{\sum(x-\bar{x})^2}{n}\right) = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$$

Unbiased estimate of common population variance from two samples:

$$s^2 = \frac{\sum(x_1-\bar{x}_1)^2 + \sum(x_2-\bar{x}_2)^2}{n_1+n_2-2}$$

Regression and correlation

Estimated product moment correlation coefficient :

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\left\{\sum(x-\bar{x})^2\right\}\left\{\sum(y-\bar{y})^2\right\}}} = \frac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \dfrac{(\sum x)^2}{n}\right)\left(\sum y^2 - \dfrac{(\sum y)^2}{n}\right)}}$$

Estimated regression line of y on x :

$$y - \bar{y} = b(x-\bar{x}), \quad \text{where } b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$