

Graphical Transformations



To transform the graph of $y = f(x)$ into the graph of $y = 3 + 5f(2x - 1)$:

$$y = f(x) \rightarrow y = f(x - 1) \rightarrow y = f(2x - 1) \rightarrow y = 5f(2x - 1) \rightarrow y = 3 + 5f(2x - 1)$$

| | Algebraic Implications | Result | Geometric Descriptions |
|----------|-----------------------------------|----------------------|--|
| 1 | Replace x by $(x - 1)$ | $y = f(x - 1)$ | Translation of 1 unit to the right. |
| 2 | Replace x by $2x$ | $y = f(2x - 1)$ | Scaling parallel to the x -axis by a factor of $\frac{1}{2}$. |
| 3 | Multiply 5 to all y coordinates | $y = 5f(2x - 1)$ | Scaling parallel to the y -axis by a factor of 5. |
| 4 | Add 3 to all y coordinates | $y = 3 + 5f(2x - 1)$ | Translation of 3 units in the direction of the positive y -axis. |

| | Algebraic Implications | Geometric Descriptions |
|--|--|--|
| $y = f(x) + 3$ [$(y - 3) = f(x)$] | Add 3 to all y values [Replace all y by $(y - 3)$] | Translation of 3 units in the direction of the positive y -axis. |
| $y = f(x) - 4$ [$(y + 4) = f(x)$] | Subtract 4 from all y values [Replace all y by $(y + 4)$] | Translation of 4 units in the direction of the negative y -axis. |
| $y = f(x + 3)$ | Replace all x by $(x + 3)$ | Translation of 3 units in the direction of the negative x -axis. |
| $y = f(x - 4)$ | Replace all x by $(x - 4)$ | Translation of 4 units in the direction of the positive x -axis. |
| $y = 3f(x)$ | Multiply all y values by 3 | Scaling parallel to the y -axis by a factor of 3. |
| $y = f(4x)$ | Replace all x values by $4x$ | Scaling parallel to the x -axis by a factor of $\frac{1}{4}$. |
| $y = -f(x)$ | Multiply -1 to all y values | Reflection in the x -axis. |
| $y = f(-x)$ | Replace all x by $-x$ | Reflection in the y -axis. |