

$$\begin{aligned}
 Q4) \quad & (8+x)^{\frac{1}{3}} \\
 &= 8^{\frac{1}{3}} \left(1 + \frac{x}{8}\right)^{\frac{1}{3}} \\
 &= 2 \left(1 + \frac{1}{3}\left(\frac{x}{8}\right) + \frac{\frac{1}{3}(-\frac{2}{3})}{2} \left(\frac{x}{8}\right)^2 + \dots\right) \\
 &= 2 \left(1 + \frac{x}{24} - \frac{x^2}{516} + \dots\right), \quad |x| < 8.
 \end{aligned}$$

Given x is sufficiently small,

$$\begin{aligned}
 & \frac{1}{\cos 2x} \\
 & \approx \frac{1}{1-2x^2} \\
 & \approx (1-2x^2)^{-1} \\
 & \approx 1 + (-1)(-2x^2) + \dots \\
 & \approx 1 + 2x^2 + \dots, \quad |x| < \frac{1}{\sqrt{2}}
 \end{aligned}$$

$1-2x^2 < 1$
 $2x^2 < 1$
 $|x^2| < \frac{1}{2}$
 $|x| < \frac{1}{\sqrt{2}}$

$$\frac{(8+x)^{\frac{1}{3}}}{\cos 2x}$$

$$\begin{aligned}
 &= \left(2 + \frac{x}{12} - \frac{x^2}{516} + \dots\right) (1 + 2x^2 + \dots) \\
 &= 2 + \frac{x}{12} - \frac{x^2}{516} + 4x^2 + \dots = 2 + \frac{x}{12} + \frac{115}{288}x^2, \quad |x| < \frac{1}{\sqrt{2}} \quad *
 \end{aligned}$$

$$\begin{aligned}
 Q5: \quad & \ln(1+2x) \\
 &= (2x) - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \dots \\
 &= 2x - 2x^2 + \frac{8x^3}{3} - \dots, \quad -\frac{1}{2} < x \leq \frac{1}{2} \quad *
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad & ax(1+bx)^c \\
 &= ax \left[1 + c(bx) + \frac{c(c-1)}{2} (bx)^2 + \dots\right] \\
 &= ax \left[1 + bcx + \frac{c(c-1)}{2} b^2x^2 + \dots\right] \\
 &= ax + abcx^2 + \frac{ab^2c(c-1)}{2} x^3 + \dots
 \end{aligned}$$

Comparing x term,

$$\begin{aligned}
 ax &= 2x \\
 a &= 2 \quad *
 \end{aligned}$$

Comparing x^2 term,

$$\begin{aligned}
 abc &= -2 \\
 2bc &= -2 \\
 bc &= -1 \\
 b &= -\frac{1}{c}.
 \end{aligned}$$

Comparing x^2 term

$$\begin{aligned}
 \frac{ab^2c(c-1)}{2} &= \frac{8}{3}. \\
 \left(-\frac{1}{c}\right)^2 c(c-1) &= \frac{8}{3} \\
 1 - \frac{1}{c} &= \frac{8}{3} \\
 \frac{1}{c} &= -\frac{5}{3} \\
 c &= -\frac{3}{5} \quad * \\
 \Rightarrow b &= \frac{5}{3} \quad *
 \end{aligned}$$

$$ax(1+bx)^c$$

$$= 2x \left(1 + \frac{5}{3}x\right)^{-3/5}$$

The x^4 term is

$$2x \frac{(-3/5)(-8/5)(-13/5)}{3!} \left(\frac{5}{3}x\right)^3$$

$$= -\frac{104}{27} x^4 \quad *$$