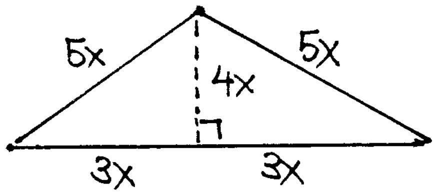
2(i). Area of rectangular sheet

$$(10x+2y)(30) = 1500$$

$$10x+2y = 50$$

$$y = 25-5x$$



Volume of the letterbox

$$= \left[\tfrac{1}{2}(6x)(4x) + 6x(y)\right] \times 30$$

$$= \left[12x^2 + 6xy\right] \times 30$$

$$= 360x^2 + 180x\,[25-5x]$$

$$= 4500x - 540x^2 \quad \text{(shown)}$$

(ii).

$$V = 4500x - 540x^2$$

$$\frac{dV}{dx} = 4500 - 1080x$$

At the stationary point, $\frac{dV}{dx} = 0$

$$4500 - 1080x = 0$$

$$x = 25/6 \text{ cm} \quad \#$$

$$\frac{d^2V}{dx^2} = -1080 < 0$$

Since $\frac{d^2V}{dx^2} < 0$, V is maximum when $x = \frac{25}{6}$.

maximum volume, $V = 4500\left(\frac{25}{6}\right) - 540\left(\frac{25}{6}\right)^2$

$$= 9375 \text{ cm}^3 \quad \#$$

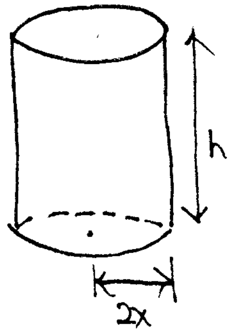
3 Volume of can

$= \pi (2x)^2 h$

$= 4x^2 \pi h.$

$\therefore \quad 4x^2 \pi h = 100\pi$

$h = \dfrac{100\pi}{4x^2\pi} = \dfrac{25}{x^2}.$



Surface area of a can, S

$= 2\left[\pi(2x)^2\right] + 2\pi(2x)h.$

$= 8\pi x^2 + 4\pi x\left(\dfrac{25}{x^2}\right)$

$= 8\pi x^2 + \dfrac{100\pi}{x}$ (shown).

To minimise production cost, the surface area of a can has to be minimum.

$\dfrac{dS}{dx} = 16\pi x - \dfrac{100\pi}{x^2}$

At the stationary point, $\dfrac{dS}{dx} = 0$

$16\pi x = \dfrac{100\pi}{x^2}.$

$x^3 = \dfrac{100}{16}. \Rightarrow x = \sqrt[3]{\dfrac{25}{4}}$ or 1.842 cm.

$\dfrac{d^2S}{dx^2} = 16\pi + \dfrac{200\pi}{x^3}$

When $x = \sqrt[3]{\dfrac{25}{4}}$, $\dfrac{d^2S}{dx^2} = 16\pi + \dfrac{200\pi}{\left(\frac{25}{4}\right)} > 0$

When $x = \sqrt[3]{\dfrac{25}{4}}$, S is of the smallest value.

minimum $S = 8\pi\left(\dfrac{25}{4}\right)^{2/3} + \dfrac{100\pi}{\left(\frac{25}{4}\right)^{1/3}}$

$= 85.274 + 170.5539$

$= 255.828 \text{ cm}^2.$

The minimum production cost

$= \dfrac{255.828}{100} \times 3 = 7.67$ cents