Summation

Given that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ and $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$, find the sum of the first n terms of the series (1)(2)(4) + (2)(3)(5) + (3)(4)(6) + … [4]

Solution	**Walkthrough Guide**
(1)(2)(4) + (2)(3)(5) + (3)(4)(6) + … $= \sum_{r=1}^{n} r\,(r+1)(r+3)$	Express the given series in the sigma notation. **M1**
$= \sum_{r=1}^{n} r^3 + 4r^2 + 3r$ $= \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} 4r^2 + \sum_{r=1}^{n} 3r$	Open the brackets $\sum_{r=1}^{n}(r^3 \pm 4r^2) = \sum_{r=1}^{n} r^3 \pm \sum_{r=1}^{n} 4r^2$ **M1**
$= \sum_{r=1}^{n} r^3 + 4\sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r$	$\sum_{r=1}^{n} 4r^2 = 4\sum_{r=1}^{n} r^2$
$= \frac{1}{4}n^2(n+1)^2 + 4\left[\frac{1}{6}n(n+1)(2n+1)\right] + 3\left[n\frac{(1+n)}{2}\right]$	Apply the given formulae. $\sum_{r=1}^{n} r$ is an AP, recall the formula. **M1**
$= \frac{1}{4}n^2(n+1)^2 + \left[\frac{2}{3}n(n+1)(2n+1)\right] + \left[\frac{3}{2}n(n+1)\right]$ $= n(n+1)\left[\frac{1}{4}n(n+1) + \frac{2}{3}(2n+1) + \frac{3}{2}\right]$	Simplify the answers Take out the common factor $n(n+1)$. It is pure algebraic manipulation.
$= n(n+1)\left[\frac{1}{4}n^2 + \frac{19}{12}n + \frac{13}{16}\right]$	It is a bit untidy here…but on certain occasions, the final answer is a nice expression.
$= \frac{1}{4}n^4 + \frac{22}{12}n^3 + \frac{115}{48}n^2 + \frac{13}{16}n$	**M1**